Sky Brightness Monitor Notes

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1. Introduction: Sky Brightness.

We intend to measure sky brightness for the g,r,i,z and y bands. Table 1 gives typical sky brightness values for a good astronomical site, in Johnson (Vega-based) magnitudes per square arcsecond. The goal of this document is to explore, for a single-pixel (filtered photodiode) detector and a fixed etendue (FOV x collection aperture) whether it's better to use a small aperture and a wide field or a large aperture and small footprint on the sky. The etendue determines the total sky flux seen by the detector. A good compendium of data on sky characteristics is http://www.cfht.hawaii.edu/Instruments/ObservatoryManual/CFHT_ObservatoryManual_(Sec_2).html.

This is somewhat a stream-of-consciousness document, and apologies are offered to the reader for lack of organization and coherence.

Days after New moon	B	V	R	I
0	22.7	21.8	20.9	19.9
3	22.4	21.7	20.8	19.9
7	21.6	21.4	20.6	19.7
10	20.7	20.7	20.3	19.5
14	19.5	19.5	19.9	19.2

Table 1.
Typical sky brightness values for a good astronomical site (CTIO). Units are Vega-based magnitudes per square arcsecond.

We will adopt the utilitarian definition of "sky brightness" as being the component of detected flux that is (at least locally) uniform in surface brightness. A variety of sources contribute to this sky brightness, and the dominant term varies with wavelength. Sky in the bluer wavelengths depends on moon phase to due the wavelength dependence of Rayleigh and aerosol scattering. For wavelengths longer than about 7400A emission from OH molecules wins, and the sky brightness is dominated by sharp emission lines. Unresolved celestial sources, scattered starlight, and zodiacal light also contribute.

The aspects of sky brightness that one would imagine diminishing with altitude are those associated with the atmosphere, namely scattering (both Rayleigh and aerosol), and emission (if we get above the OH and O2 emission regions). This motivates measurements in both the NIR and blue regions of the spectrum. One might imagine achieving V band sky brightness that is considerably less sensitive to moon phase, due to reduced atmospheric scattering along the line of sight, for example.

2. Estimate of Flux Contamination from Discrete Sources.

In order to consider the contamination effects from bright stars for a sky brightness measurement that spans a solid angle Ω, we can ask how large an angle of sky must be integrated over in order to have a flux equal to that from the point source. Over this angle the sky brightness would be overestimated by a factor of 2. A good reference for star counts as a function of Galactic latitude and longitude is Bahcall's Annual Reviews article, "Star Counts and Galactic Structure" (ARA&A, 24, 577, 1986). Star counts of course reach their minimum at the Galactic poles, and Figure 1 (which is Figure 2 in the Bahcall paper) shows differential stellar density at the North Galactic pole. More contemporary versions of the star counts paper do exist, but this will do fine for current purposes.

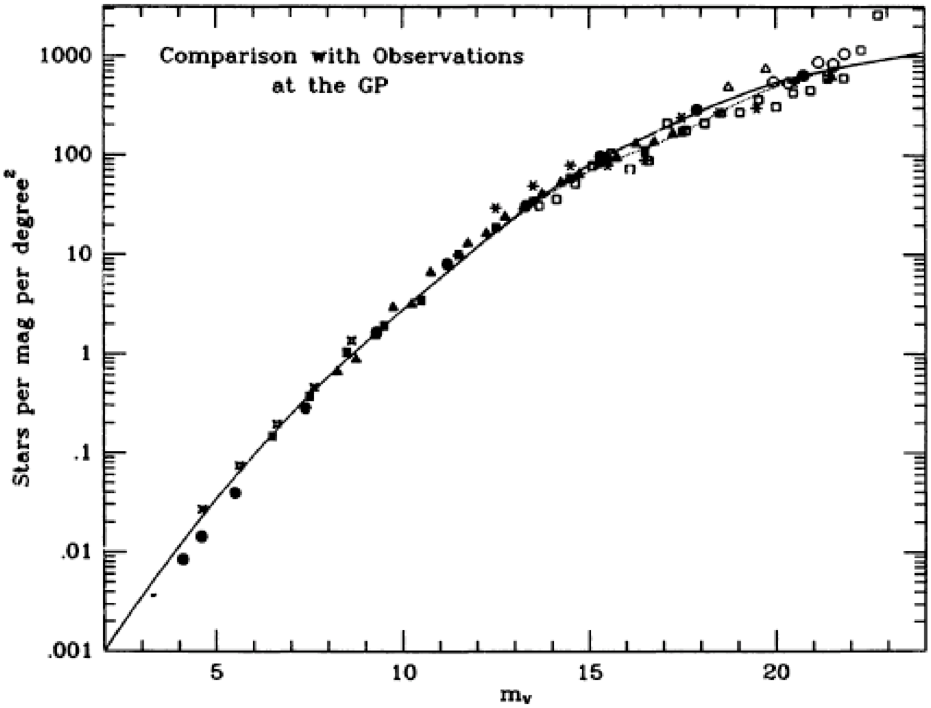

Figure 2 The north galactic pole. Differential star counts per magnitude per square degree for the galactic pole. The solid curve is predicted by the standard model. Data from Seares et al. (1925) as reduced to the visual band in Paper I are plotted as filled circles; data from Weistrop (1972) with Faber et al. (1976) corrections as filled squares; data from McLaughlin (1983) as open crosses; data from Reid & Gilmore (1983) as filled triangles; data from King (Chiu 1980) as asterisks; data from Jarvis & Tyson (1981) as open squares; data from Peterson et al. (1979) as open triangles; and data from Kron (1978, 1980) as open circles. This figure is taken from Paper II.

Figure 1. This figure shows differential star counts at the North Galactic pole, and is taken from Bahcall (1986, his Figure 2).

We'll consider the V band in evaluating the optimal FOV considerations, as these are magnitudes Bahcall provides. The B band sky brightness is comparable to that in V.

Furthermore this is the conservative approach since the sky in redder bands is much brighter, easing our stellar contamination problem. The sky brightness in V does vary with lunar phase, and during dark time is about 22nd mag per sq arc sec. The sky area needed to have sky flux equal to the stellar flux from a single star with V magnitude = m is then given by A (sq arcsec) = 10^(22 – m)/2.5, so that log10(A) = (22 – m)/2.5. Figure 2 is a plot of the sky area needed to have a detected V band sky dark time flux equal to that of a contaminating star.

An on-line web page that purports to calculate Galactic star counts for different lines of sight, based on the Bahcall-Soniera Galactic model is http://oalm.astronomia.edu.uy/star_count/Star_Counts.htm.

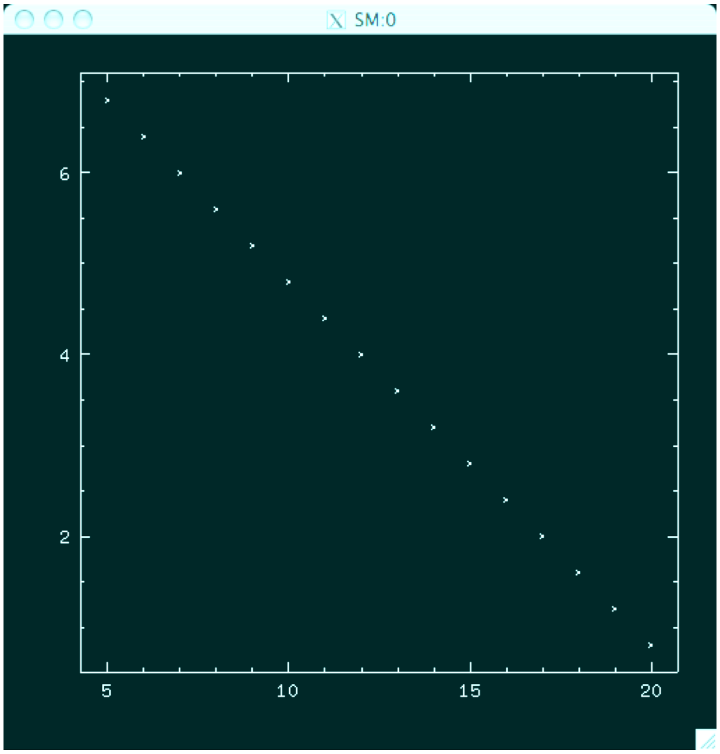

Figure 2. Sky area integration needed to have sky brightness equal to a contaminating star's flux. The x axis is V band magnitude, the y axis is log(A) where A is in square arc seconds. One square degree has log(A) = 7.1. Demanding that the single contaminating star be limited to 10% of the total flux would add one unit to the y axis. Even a V=7 star would only contribute 10% of the apparent sky flux in a one square degree FOV.

In an attempt to build some intuition, let's consider a single pixel that subtends one square degree. Based on Figure 1 we'd expect on average one star at V=10 in this field of view, at the North Galactic Pole (NGP). Now looking at Figure 2, the flux from this one star is one tenth of that from the sky once we look at more than 10^6 square arc seconds, which is equal to 0.08 square degrees (I've added one unit to the value at V =10 from Figure 2 in order to achieve <10% flux contamination.)

So now examine the other limit: a 10 sq arcsec pixel. The sky flux in V would be equivalent to 10 stars of 22nd magnitude, or a single star of V=19.5. We could tolerate a 22nd magnitude star and still have less then 10% stellar flux contamination. So what's the probability of having such a star in the field? At the NGP Bahcall gives stellar density at 22nd as being around 1000 per square degree, so with our tiny trial field of 7.7 E -7 square degrees the likelihood of significant stellar contamination is vanishingly small. The *integral* of the luminosity function down to 22nd magnitude only produces about 2400 stars per square degree, and the chance of finding *any* star brighter than V=22 in a 10 sq arcsec pixel is only around 2×10^{-4}.

I think this simple comparison argues in favor of smaller fields of view. The engineering problem, however, is that this implies a long focal length and at fixed f-number (to keep the same photon flux incident on the detector) a very large aperture optical system. So the issue is how big a field can we get away with.

The above was of course a naïve analysis. A full consideration would consider the Poisson distribution of stars in all magnitude bins, at different Galactic coordinates, and the tradeoff of stellar flux with FOV. One aspect of this is to ask how much stellar flux arises from stars of different brightness. Does the flux from a few bright stars dominate over the more numerous fainter ones? I used the on-line stellar counts tool mentioned above to determine stellar counts at galactic longitude l = 176 degrees and latitude b = 55 degrees, so this corresponds to pointing away from the Galactic center and out of the plane. Table 2 shows the corresponding star counts and their respective contributions to total flux seen (on average) from a patch of sky.

V mag	Stars per sq deg per mag	Total stars per sq degree brighter than V	Flux rel to V=10th, from stars in this bin	Total flux
10	1	1	1.0	1.0
11	4	5	1.6	2.6
12	9	12	1.1	3.7
13	21	27	0.9	4.6
14	42	57	0.7	5.4
15	77	112	0.6	6.0
16	130	208	0.4	6.3
17	200	363	0.2	6.6
18	287	594	0.1	6.7
19	385	913	0.08	6.8
20	484	1326	0.04	6.8
21	579	1812	0.02	6.8
22	670	2373	0.01	6.8
Sky @ 22nd	13E6	13E6	200	200

Table 2. Flux contributions from different sections of the stellar luminosity function. The columns are (1) V magnitude, (2) slope of the local luminosity function, (3) integral of luminosity function down to given a magnitude, (4) incremental added flux relative to single V=10 star for this $\Delta m=1$ magnitude bin, and (5) total flux from all stars integrated down to a given V magnitude. Half the stellar light comes from stars brighter than 12th magnitude, on average. The final row shows the expected contribution from sky, at 22nd magnitude per square arc second, for a 1 square degree FOV.

The last row in Table 2 calculates the sky flux for 1 square degree, which corresponds to the flux from 13 million stars at V=22. The total contribution from the sky is 200/6.8 ~ 30 times brighter than the flux from *all* the stars typically contained in such field.

The Poisson statistics for V=10 stars, at an average density of one per sq degree, imply that the likelihood for finding one such star in field of one square degree is equal to the probability for finding zero.

The adaptive optics community has also considered the statistics for finding stars as a function of field of view. Figure 3 is taken from *Laser guide star for 3.6- and 8-m telescopes: performance and astrophysical implications,* Le Louran et al, MNRAS 295, 756 (1998). They show the chances of avoiding bright stars at low galactic latitudes, b=20.

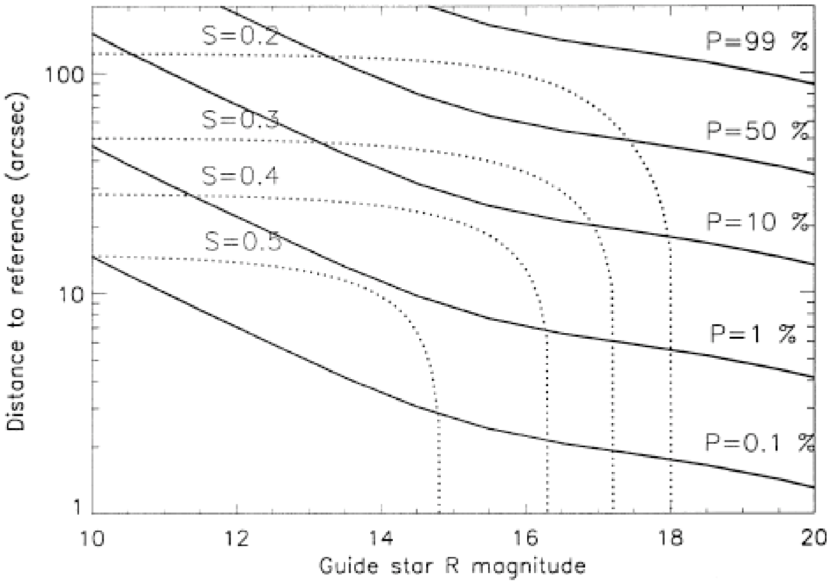

Figure 3. Likelihood of finding a star within a radius R of a random pointing, down to a given magnitude, for galactic coordinates l = 180 and b = 20. Note y axis is logarithmic.

3. Optics, f-number considerations, and flux levels.

For a fixed detector area (in sq mm) the flux intercepted from an emitting surface depends only on the f/number of the collection optics. A fast optical front end is good. If we adopt a pragmatic goal of an f/2 system, with a detector that is 1 mm on a side spanning one degree on the sky we need a focal length of FL=1mm / tan(1 deg) = 57mm, with an aperture diameter of 57/2= 28mm, or about 1 inch in diameter. This is sensible. We could probably go to f/1but let's see how this works out.

How much flux should we expect in a 1 inch aperture? A useful reference for flux levels is http://beta.physics.ucdavis.edu:8080/etc/servlets/etcHelp/ETChelpAstroSource.html

The area for our imager's input aperture is A = 615 mm^2. The exposure calculator at UC Davis claims that a 0th magnitude source gives 2 x 10^10 photons per sec per m^2 in the g band, and one tenth as many in the y band. Ignoring for the moment filter transmission and detector efficiency, the number of photons passing across the input aperture, for our one square degree system, is equivalent to having a source that is 200 times brighter than a V=10 star. This comes from the last line in Table 2, and so the sky flux corresponds to a pseudo-star with V = 10 – 2.5 log(200) ~ 4.5. This virtual object is 10^(4.5/2.5) = 60 times fainter than Vega, for 3 x 10^8 photons per sec per m^2. Our area is 615 x 10^-6 m^2, so we expect a net sky flux of 2 x 10^5 photons per second per square degree, at f/2, in the g band.

Going to f/1 would quadruple this rate. That sounds like a good idea! This would produce 8 x 10^5 sky incident photons per second per sq degree in the g band. Assuming an overall efficiency of 0.75 (for filter throughput, detector QE, etc) we should expect around 6 x 10^5 photoelectrons per second, or a photocurrent of around 10^(-13) Amps, or 0.1 pA.

Integrating this across a 100 pF capacitor would yield 1 mV per second of integrated signal. We'll certainly need an op amp that draws small (fA level) input bias currents.

Hamamatsu specs dark current for a 2 mm x 2 mm photodiode at 5 pA (S1227-33BR) at room temperature, falling by a factor of 1.12 per degree C. Putting this detector in our focal plane would increase the signal by a factor of four, with a FOV of 4 square degrees, giving about 0.5 pA of signal from the sky background. Cooling the detector by 40 C below room temperature, to T= –20C, should cut the dark current by a factor of 60, to one tenth of the signal level. Cooling by 60 C, to -40C, would cut the dark current by a factor of 100, to 0.05 pA, and this strikes me as essential otherwise we'll just have made an expensive "dark current" thermometer.

4. Conclusions.

A filtered photodiode with a field of view of a few square degrees should yield a flux that is usually comfortably dominated by the diffuse sky brightness. As long as we avoid pointing at sources brighter than V~7 the flux from discrete sources should be below 10% of the sky flux.

Our challenge is dark current, not discrete point source contamination.

A system with an f/1 lens and a focal length of ~50mm, feeding a detector 2mm x 2mm seems like a sensible choice. It is essential to cool the detector to temperatures at or below –40C to achieve the requisite sensitivity. This should deliver around 0.5 pA of photocurrent from a dark night sky.